Answer all.

1. Let \([0, 1], \mathcal{B}, m\) be the given probability space where \(\mathcal{B}\) is the Borel \(\sigma\)-algebra on \([0, 1]\) and \(m\) is the Lebesgue measure on \([0, 1]\). Set \(X : [0, 1] \to \mathbb{R}\) by \(X(\omega) = 1_{[0, 1/2]}(\omega)\), where \(1_A\) is the indicator function of the set \(A\). (i) Find out the distribution function of \(X\). (ii) Construct another random variable \(Y\) on the same probability space such that a) \(X\) and \(Y\) have same distribution, b) \(X\) and \(Y\) are independent random variables. (iii) Is it possible to construct \(Y\) such that \(\min\{X, Y\} = 0\) almost surely? Justify.

2. Let \(X\) be the distribution of outcome of a rolling unbiased dice. (i) Define a random variable of that distribution on \([0, 6], \mathcal{B}, \frac{1}{6}m\) probability space where \(\mathcal{B}\) and \(m\) are as before. (ii) Write down the \(\sigma\)-algebra generated by this random variable. (iii) Find out the distribution function, and distribution measure.

3. (i) Prove that if a distribution measure is absolutely continuous w.r.t. the Lebesgue measure, then the CDF is continuous. (ii) Would CDF be differentiable also?